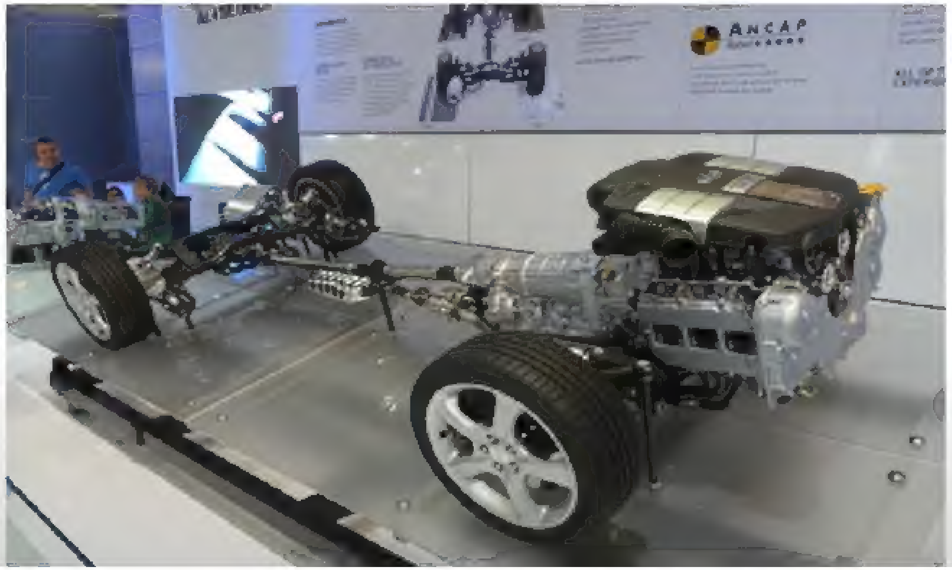
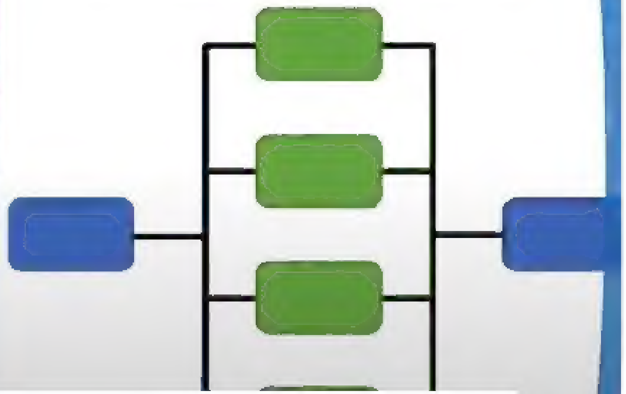
# SYSTEM RELIABILITY

One of the major activities within Reliability Engineering is understanding the **total System Reliability** for a large and **complex products** or processes.

## Series System



## Parallel System



# SERIES RELIABILITY

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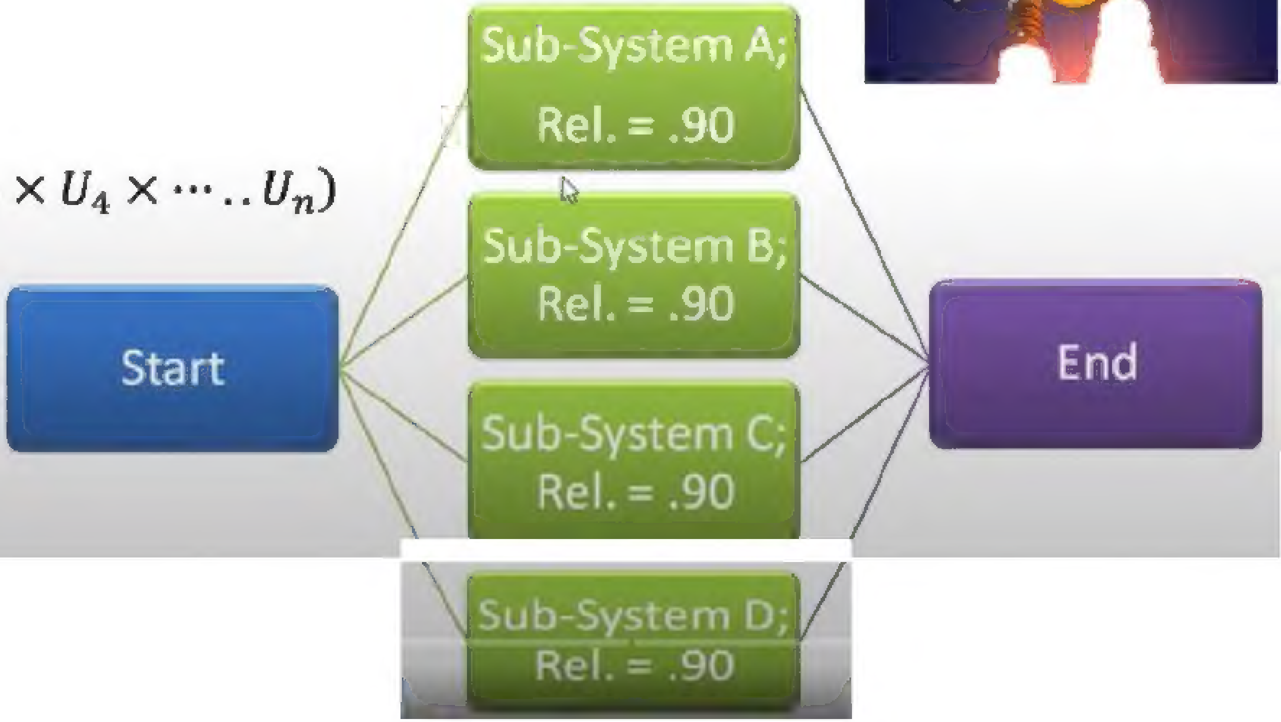
Water Inlet Valve → Electrical Controls → Pump Motor → Washer Arms → Drain Pump → Heating Element

$$Series\ System\ Reliability = R_{system} = R_1 \times R_2 \times R_3 \times \ldots \ldots R_n$$

# PARALLEL RELIABILITY

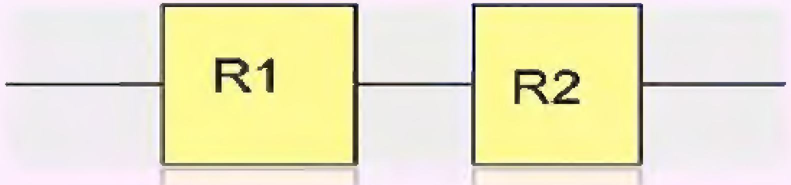
In a **Parallel System**, there are redundant pathways where ALL sub-systems must fail before the entire system reliability is impacted.

$$Parallel\ System\ Reliability = R_{system} = 1 - (U_1 \times U_2 \times U_3 \times U_4 \times \cdots ..U_n)$$



- A complex system consists of many components for performing its intended function
- For such complex systems, we can use reliability block diagrams to estimate system reliability
- In reliability block diagrams, reliability of each component is assumed to be independent

## Components in series

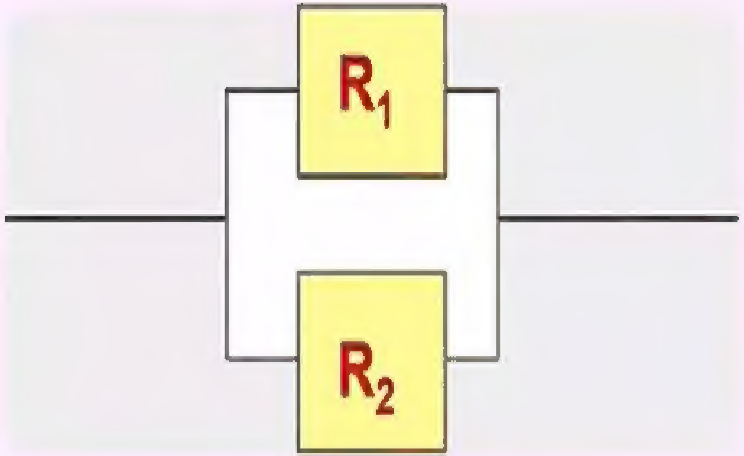



Components in Series

System reliability $R_{system} = R_1 \times R_2$

**Components in series are logical and not physical!**

# Components in parallel



Components in Parallel

System unreliability

$$(1 - R_{system}) = (1 - R_1) \times (1 - R_2)$$

**Components in parallel are logical and not physical!**

# Components in series: Application Example

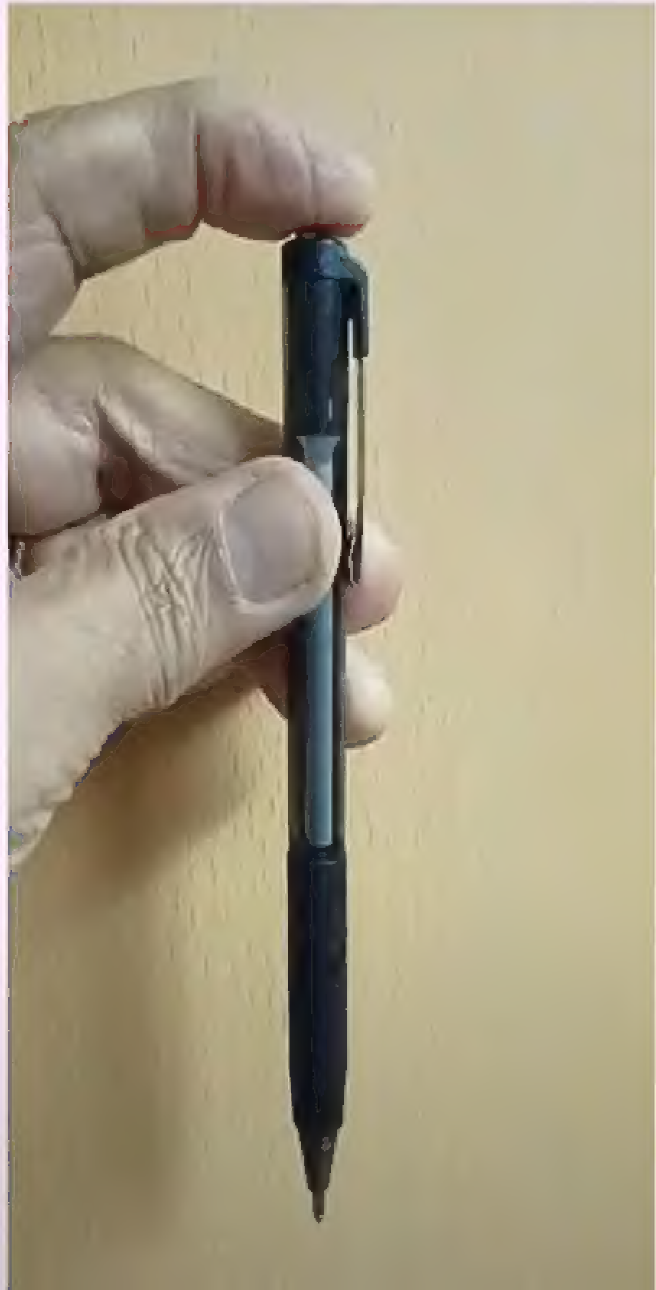
Consider a simple example of a ball pen!

We will consider three main parts for its satisfactory operation:

(1)Retracting mechanism: R1=0.9

(2)Spring : R2=0.95

(3)Ink refill : R3=0.92

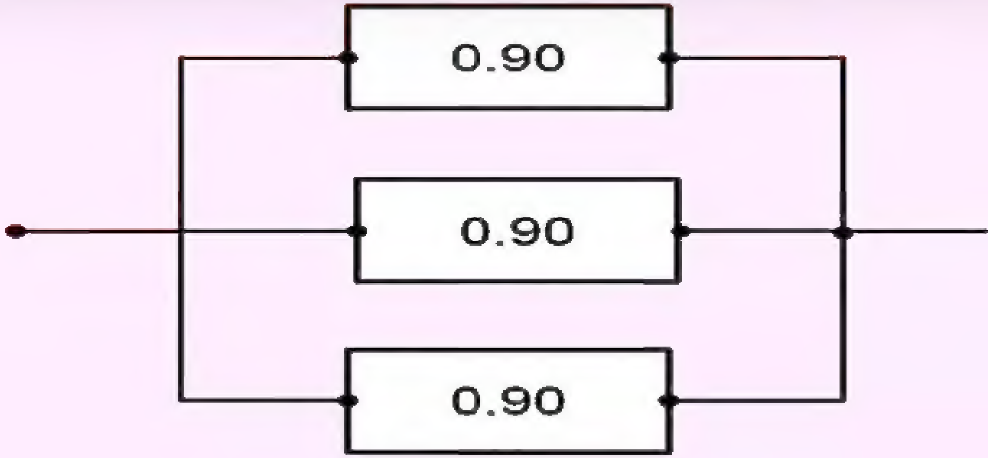


**Ball Pen Reliability:**

$$R_{system} = R_1 \times R_2 \times R_3 = 0.9 \times 0.95 \times 0.92 = 0.7866$$

# Components in parallel: Application Example

Consider that a computer has three USB ports, each with reliability of 0.9. What is the reliability of the system to connect a portable drive?



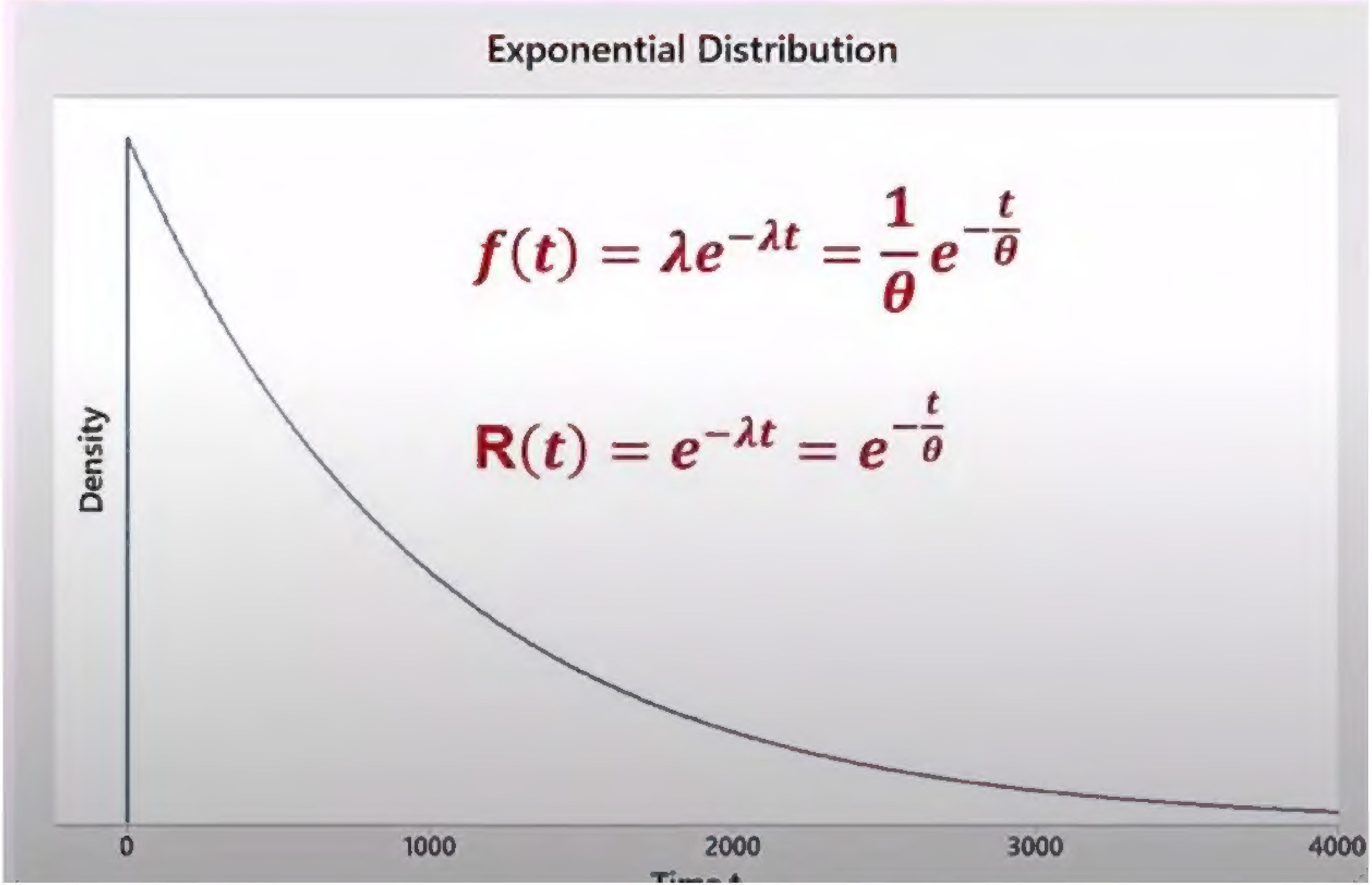

As each USB port has reliability of 0.9, unreliability of each port will be (1-0.9)=0.1.

$System\ Unreliability$:

$$(1 - R_{system}) = (1 - R_1) \times (1 - R_2) \times (1 - R_3) = 0.1 \times 0.1 \times 0.1 = 0.001.$$

$\therefore R_{system}$=1-0.001=0.999

# Constant Failure Rate (CFR)




$$f(t) = \lambda e^{-\lambda t} = \frac{1}{\theta} e^{-\frac{t}{\theta}}$$

$$\mathrm{R}(t) = e^{-\lambda t} = e^{-\frac{t}{\theta}}$$

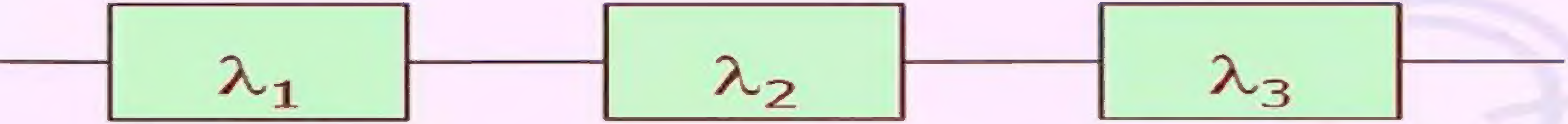
- A special case of series system is when the failure distribution for each of the subsystems is exponential i.e. failure rate or hazard rate is constant for each subsystem.
- Consider that there are n components with constant failure rates $\lambda_1, \lambda_2, \lambda_3, ...., \lambda_n$
- As each component has exponential failure distribution, system reliability can be calculated as:

$$R_{system} = R_1 \times R_2 \times \cdots \times R_n = e^{-\lambda_1 t} \times e^{-\lambda_2 t} \times \cdots \times e^{-\lambda_n t} = e^{-\{\lambda_1+\lambda_2+..+\lambda_n\}t}$$

$$R_{system} = e^{-t\sum_{i=1}^{n}\lambda_i}$$

- Thus, the system failure rate is also constant and equals the sum of the component failure rates
- This also implies that the MTBF will be $\frac{1}{\sum_{i=1}^{n}\lambda_i}$ i.e. reciprocal of the system failure rate

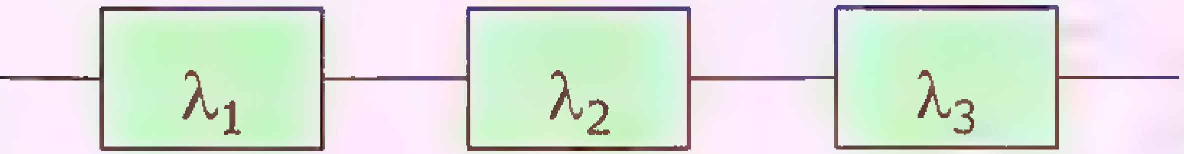
Consider that there are three components in series with failure rates as shown below. What is the reliability of the system shown at 100hrs and what is the MTBF?




$$\lambda 1 = 100 \times 10^{-6} \text{ failures/hour}$$
$$\lambda 2 = 80 \times 10^{-6} \text{ failures/hour}$$
$$\lambda 3 = 20 \times 10^{-6} \text{ failures/hour}$$

# Series Model CFR: Simple example

Consider that there are three components in series with failure rates as shown below. What is the reliability of the system shown at 100hrs and what is the MTBF?




$\lambda 1 = 100 \times 10^{-6}$ failures/hour

$\lambda 2 = 80 \times 10^{-6}$ failures/hour

$\lambda 3 = 20 \times 10^{-6}$ failures/hour

$\lambda_{System} = \lambda 1 + \lambda 2 + \lambda 3$

$\lambda_{System} = (100 + 80 + 20) \times 10^{-6}$

$\lambda_{System} = 200 \times 10^{-6}$

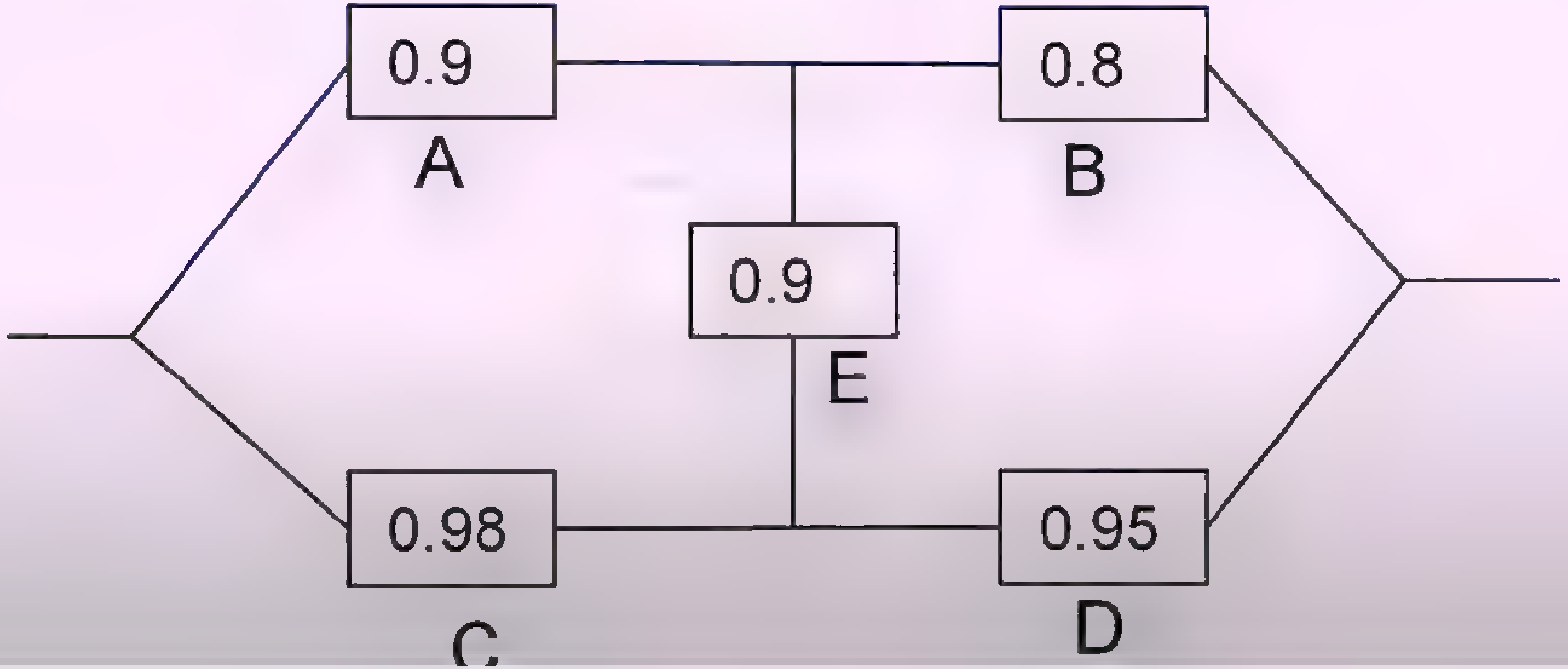
$\lambda_{System} = 0.0002$ failures/hour

$\therefore$ Reliability $R = e^{-(0.0002 x 100)} = 0.98$

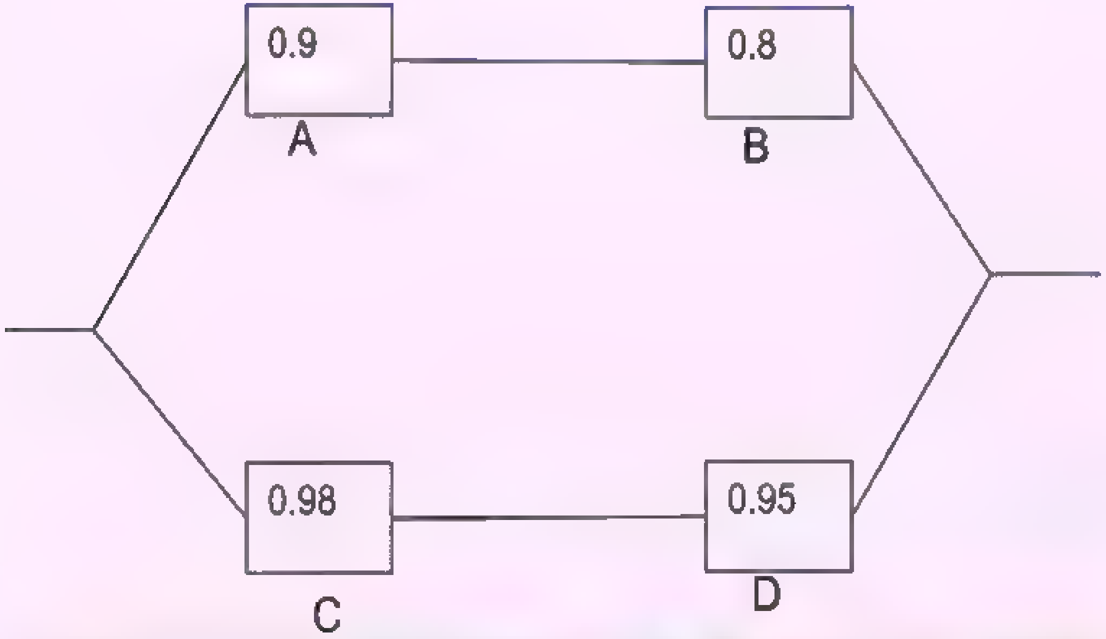
MTBF $\theta = 1/(0.0002) = 5000$ hours

# Model Using Bayes's Theorem

- In some cases, the model may neither be series nor parallel.
- In such cases Bayes's theorem can be applied.
- This is explained with the following example.
- The subsystem reliability values are shown for each component.



- **When E is in failed state, the diagram reduces to**

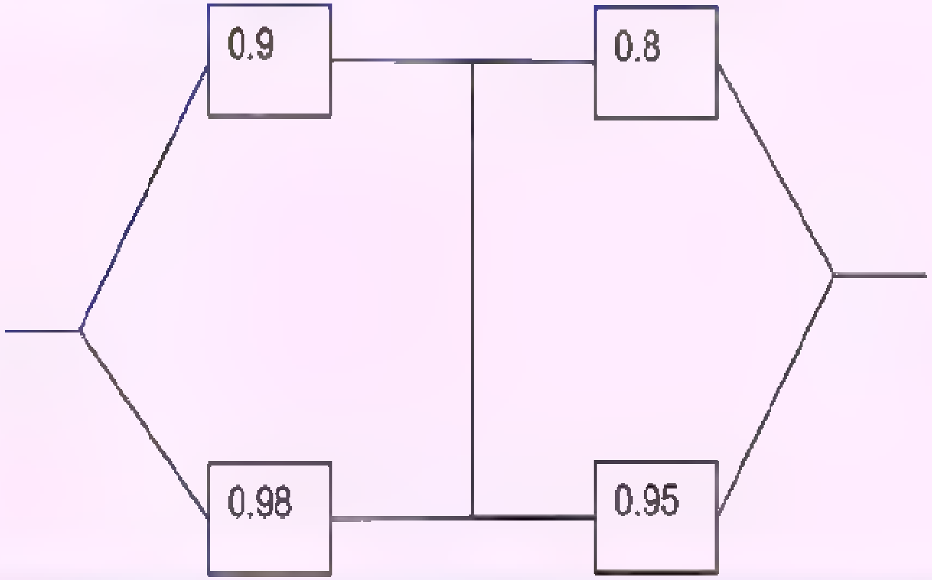



System reliability with E failed will be:

$R_{(E\ failed)}$ = 1- [1-0.9x0.8)] [1 – 0.98 x 0.95]

$R_{(E\ failed)}$ =0.98068

- **When E is in success state, the diagram reduces to**




System reliability with E working will be:

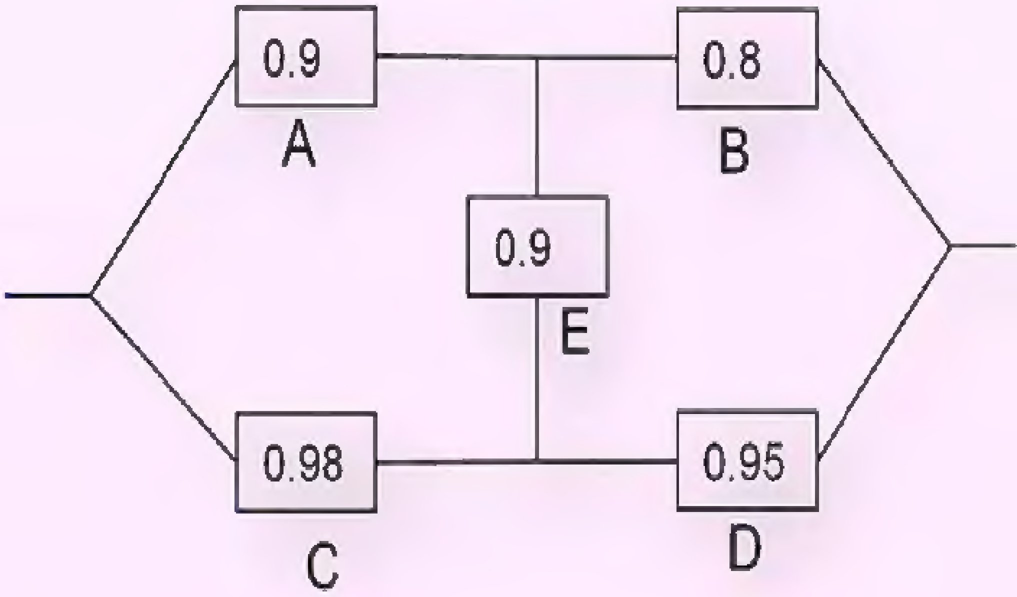
$R_{(E\ Success)}$

= (1 – 0.1x0.02)( 1 – 0.2x0.05) =0.988

The system reliability is

$$R_{System} = R_{system(E\ success)} \times P_{(E\ success)} + R_{system\ (E\ failed)} \times P_{(E)\ failed)}$$

$R_{(E\ failed)} = 0.98068$ , $R_{(E\ Success)} = 0.988$



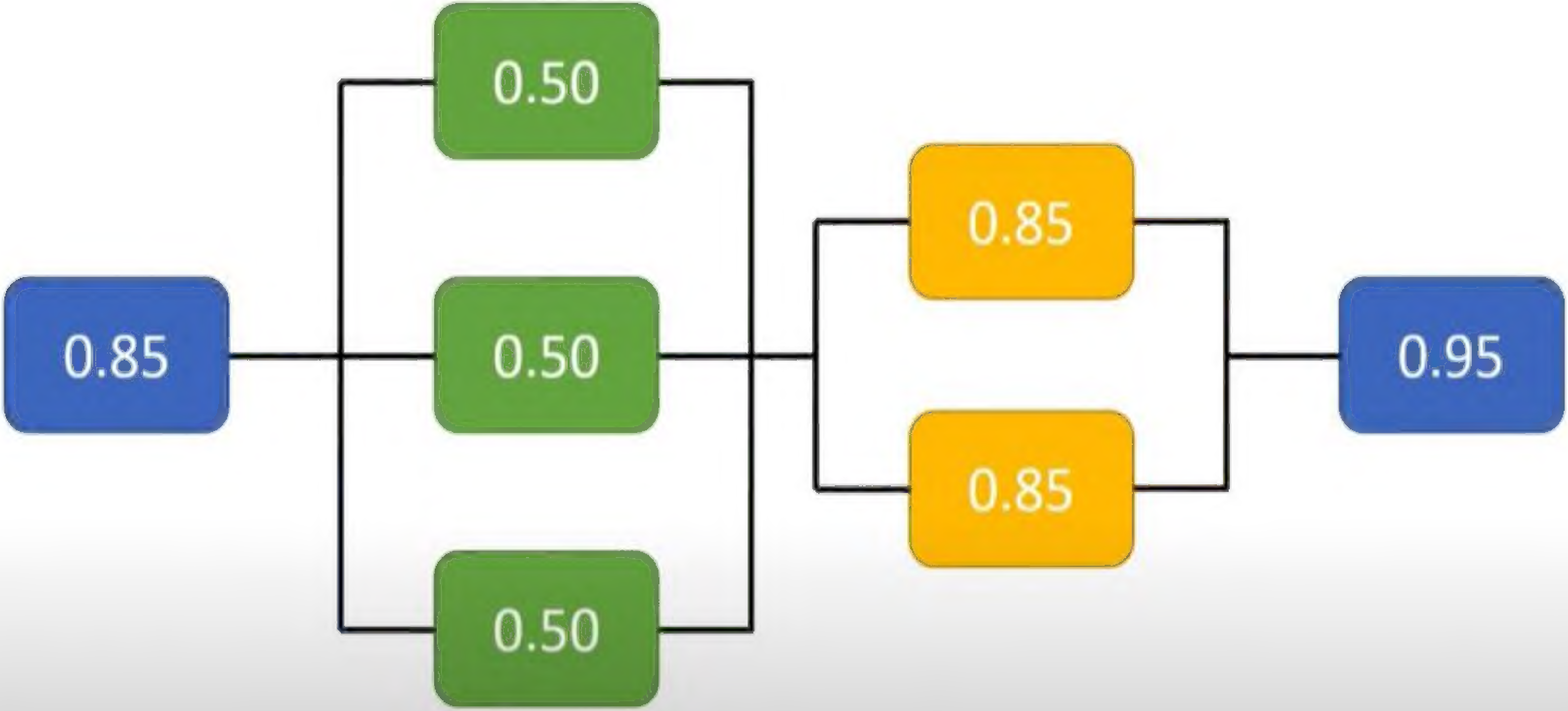

Reliability of E is 0.9 therefore probability of E in failed state is 0.1

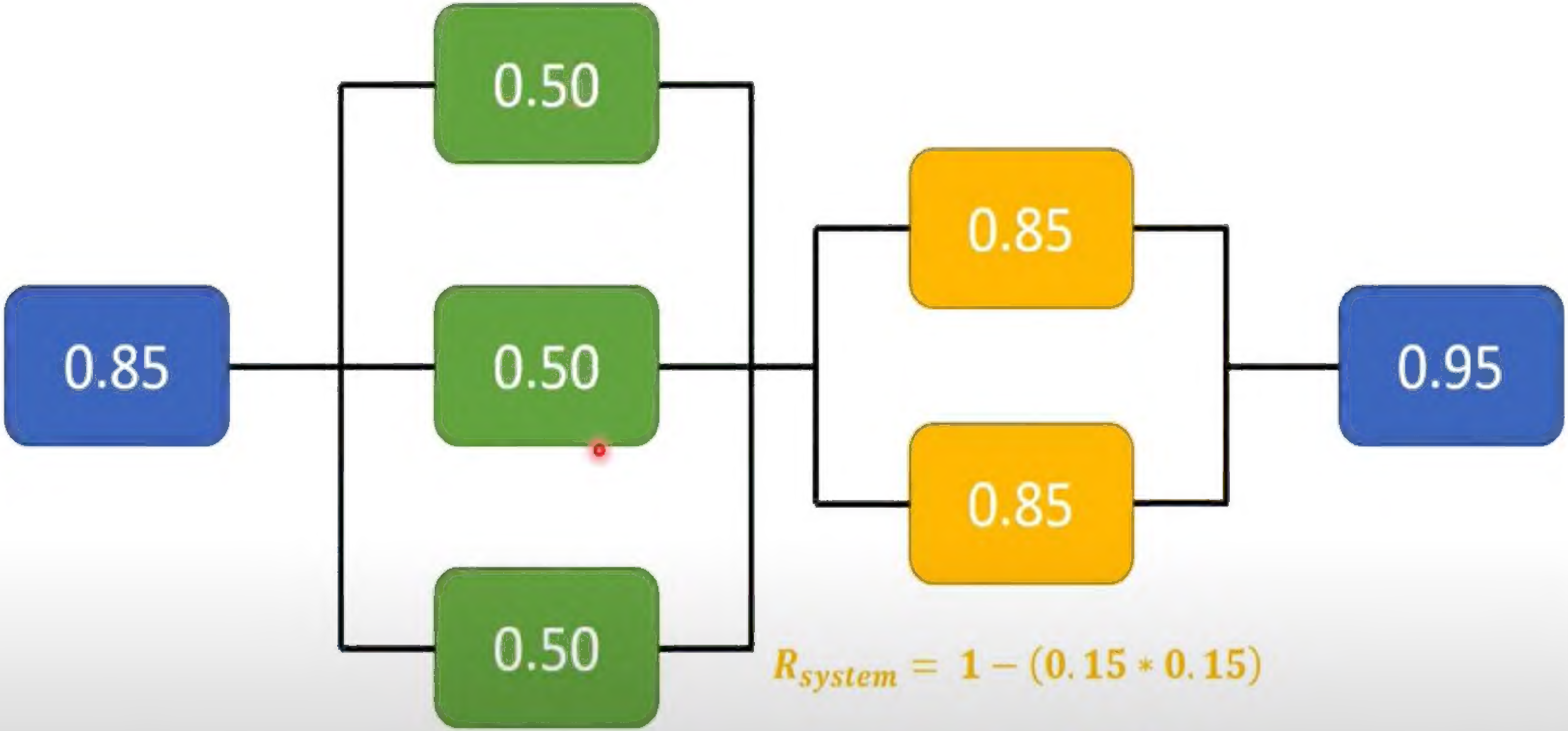
The system reliability is

$$R_{System} = R_{system(E\ success)} \times P_{(E\ success)} + R_{system\ (E\ failed)} \times P_{(E)\ failed)}$$

$$= 0.988 \times 0.9 + 0.98068 \times 0.1 = 0.9873$$

# Combined System Example RELIABILITY



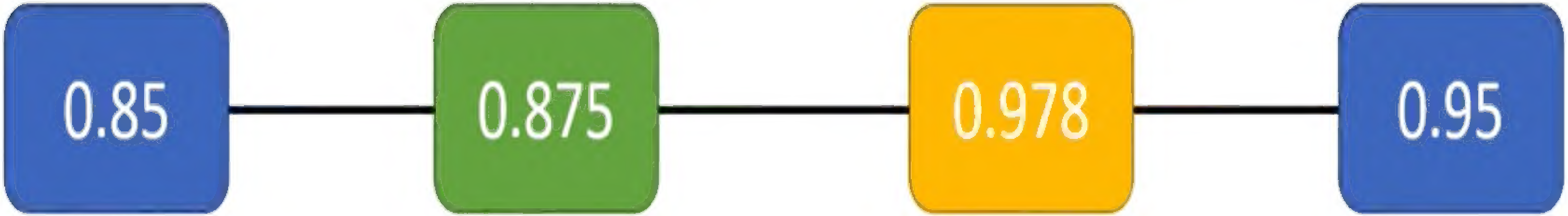



$$R_{system} = 1 - (0.15 * 0.15)$$

$$R_{system} = 0.978$$

$$R_{system} = 1 - (0.50 * 0.50 * .50)$$

$$R_{system} = 0.875$$




$$Series\ System\ Reliability = R_{system} = R_1 \times R_2 \times R_3 \times \ldots \ldots R_n$$

$$R_{system} = 0.85 * 0.875 * 0.9775 * 0.95 = 0.69$$